Summary of fundamental sessions

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Co-chaired by R.L. Dewar, T.S. Harm, H. Qin, P.J. Morrison

The aim of "fundamental" sessions

- Mathematically nontrivial & physically rational
- Foundations for transdisciplinary studies

Subjects:

1. Topological understanding of complex systems
2. Hamiltonian structures
3. Theoretical approach to nonlinear phenomena
4. Challenge of scale hierarchy
5. MHD equilibria as mathematical challenge

“topological” understanding of complex systems

How can we describe/understand complex systems *topologically*?

- “Word tree” (COT) description of 2D vortex structures and their dynamics ← T. Yokoyama
- Quantify by topological index: helicity ← M. Wheatland

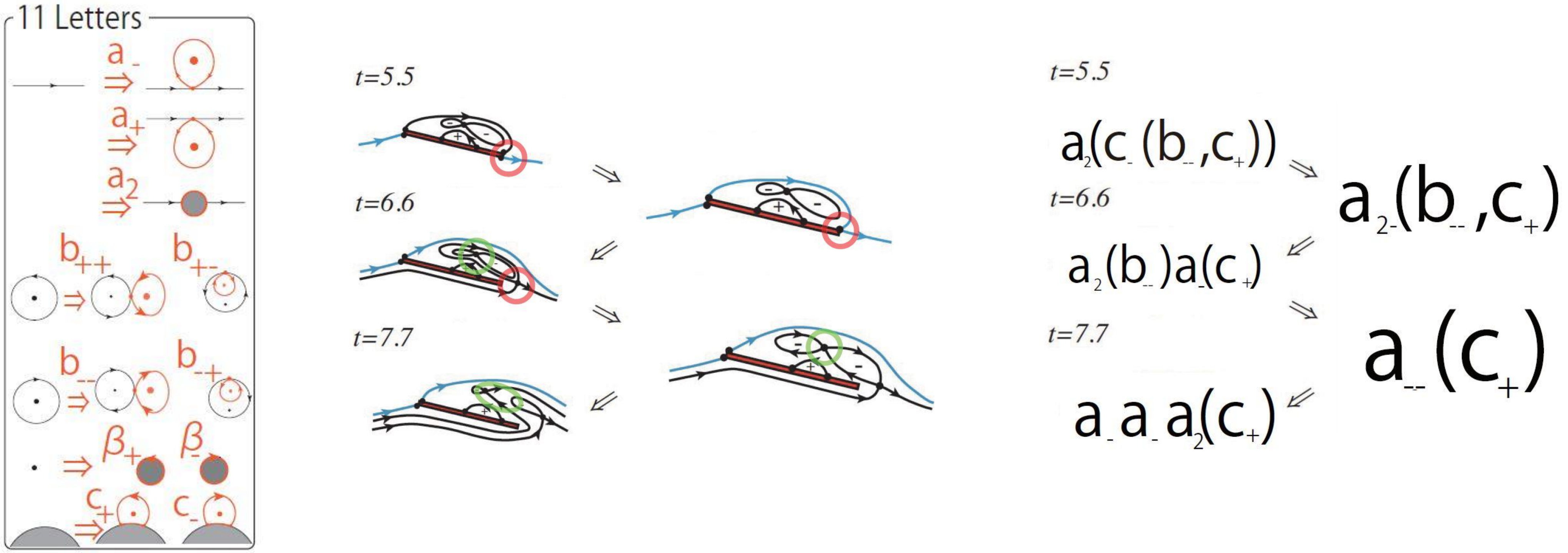
How can we find key parameters?

- Data-driven understanding by sparse modeling ← Y. Igarashi
- Bayesian inference ← M. Hole
- Deep learning for imaging diagnostics ← N. Kenmochi

“topological” understanding of complex systems (1)

COT (Cyclically Ordered rooted Tree) representation of 2D vortex structure

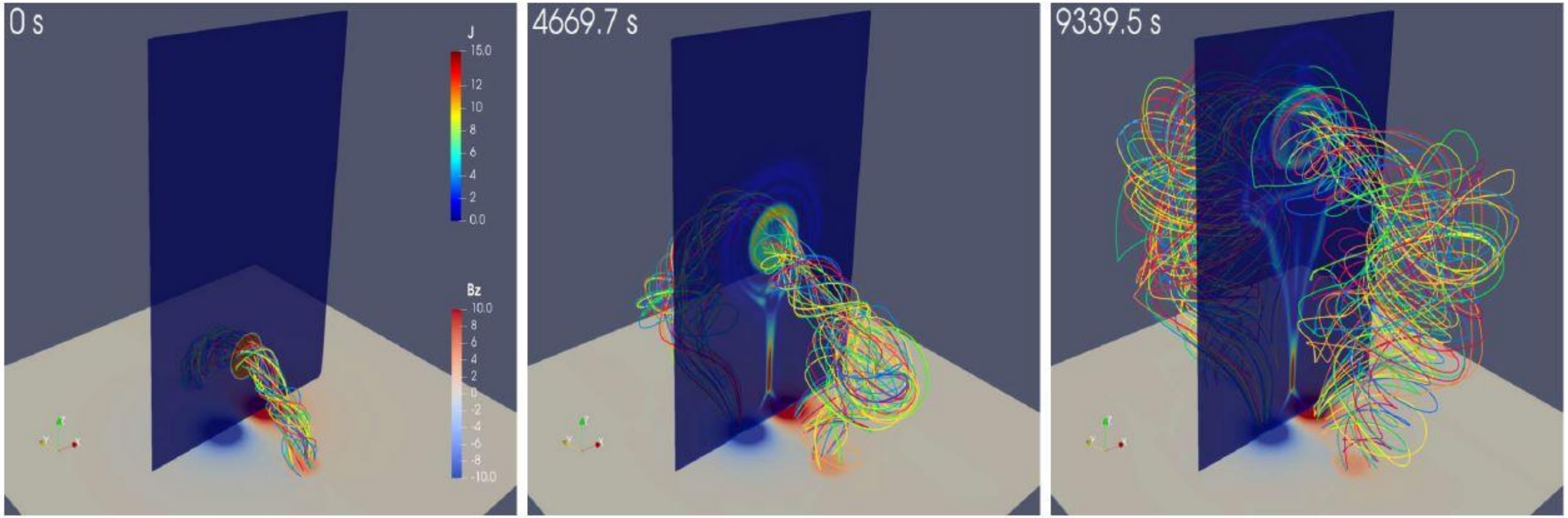
(T. Yokoyama)



“topological” understanding of complex systems (2)

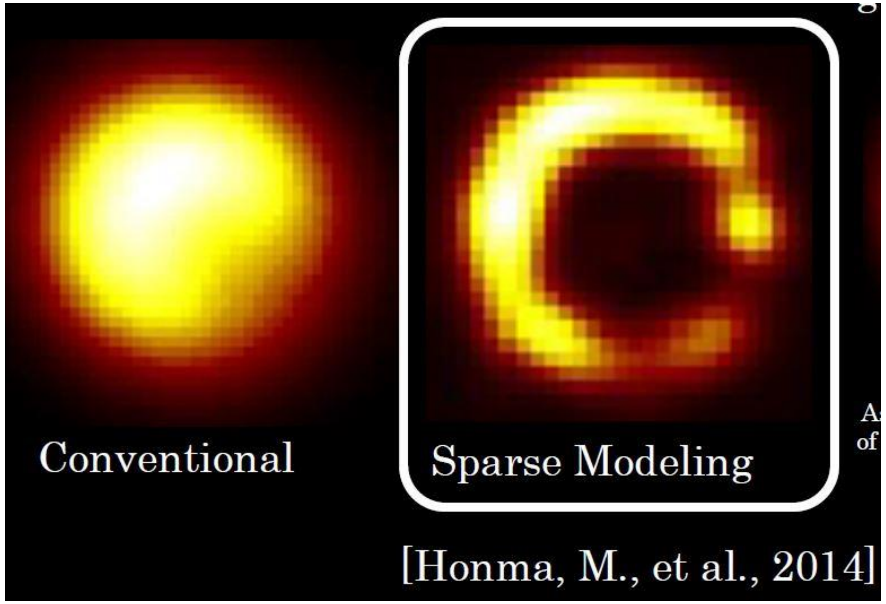
Helicity as the topological index of “twisted” fields (M. Wheatland)

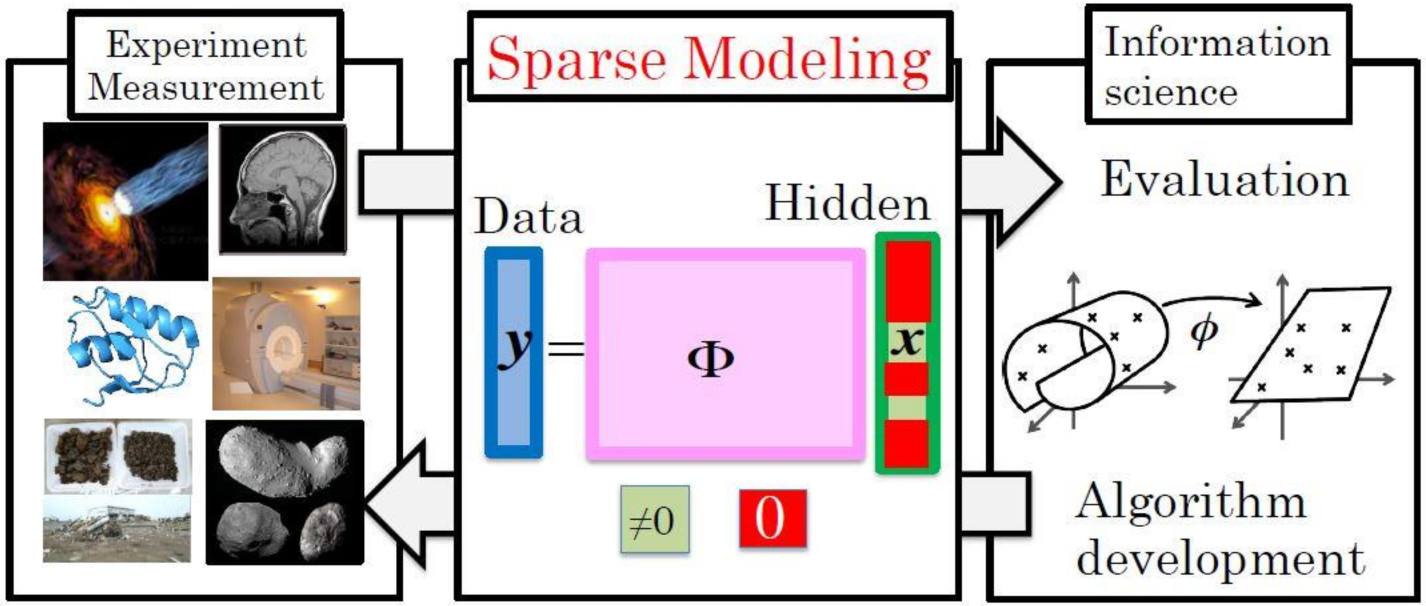
Open field needs careful treatments



“topological” understanding of complex systems (3)

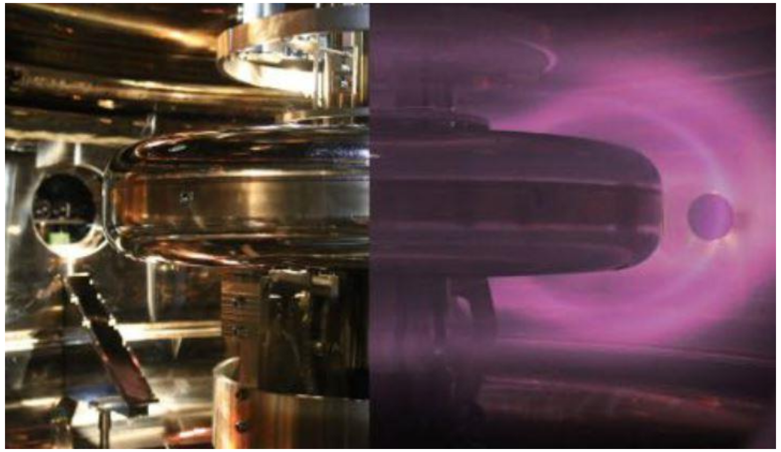
Extracting the key parameters: Sparse modeling for a data-driven approach (Y. Igarashi)

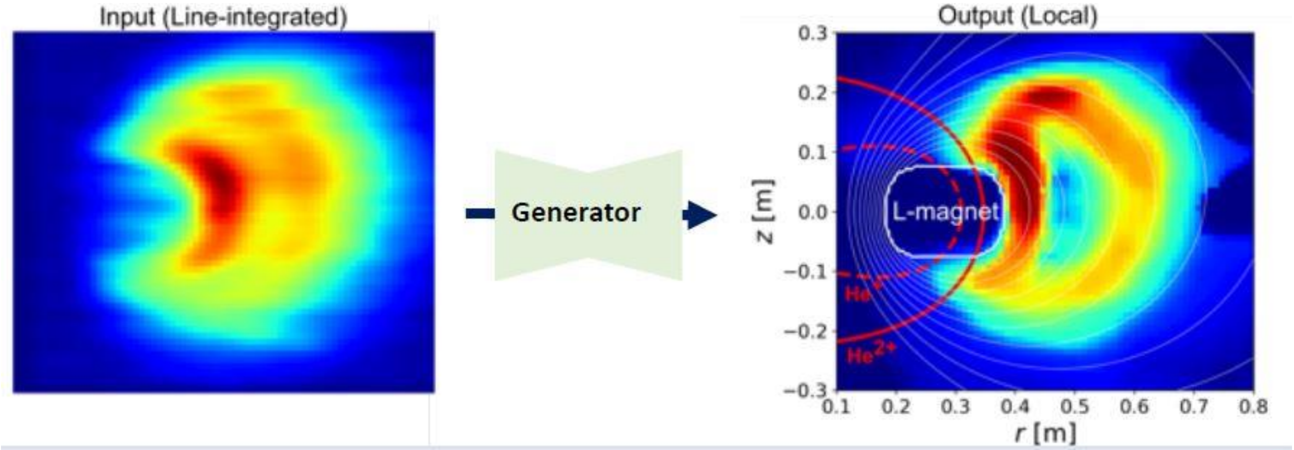




“topological” understanding of complex systems (4)

Extracting the key parameters: Machine learning to imaging diagnostics (N. Kenmoshi)





Hamiltonian structure

--- as a guide to characterize dynamics/structures

Casimirs to characterize dynamics/structures

- Noncanonical Hamiltonian mechanics and Casimirs ← P.J. Morrison
- Foliation: Isovortical → Isomagnetovortilcal tested by MRI ← R. Zou
- Constraints determining the minimum zonal enstrophy ← H. Aibara
- Kinetic effects viewed by fluid Casimirs ← R. Numata
- Deformed Li-Poisson algebra producing chirality ← Z. Yoshida
- Action principle for relativistic extended MHD ← Y. Kawazura

Modified Hamiltonian systems to relax "too many constraints" in ideal mechanics

- Metriplectic dynamics to cause relaxation of Casimir ← P.J. Morrison
- Simulated annealing to generate MHD equilibria ← M. Furukawa
- MRxMHD connecting Beltrami fields to generate 3D MHD equilibria
  ← R.L. Dewar, D. Malhotra, Z. Qu

Noncanonical Hamiltonian system

Foliation of phase space by noncanonical bracket (P.J. Morriosn)

Sophus Lie (1890)

Noncanonical Coordinates:

$$\dot{z}^\alpha = J^{\alpha\beta}\frac{\partial H}{\partial z^\beta} = \{z^\alpha, H\}, \quad \{f,g\} = \frac{\partial f}{\partial z^\alpha} J^{\alpha\beta}(z) \frac{\partial g}{\partial z^\beta}, \quad \alpha,\beta = 1,2,\ldots M$$

Poisson Bracket Properties:

antisymmetry $\longrightarrow$ $\{f,g\} = -\{g,f\}$,

Jacobi identity $\longrightarrow$ $\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$

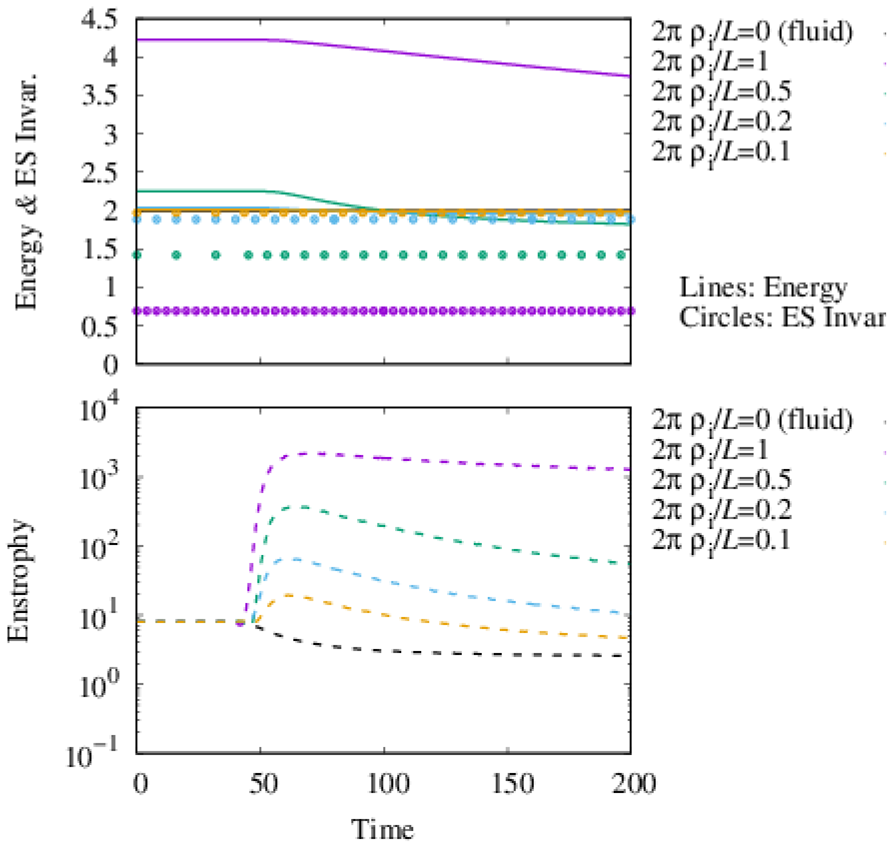
G. Darboux: $detJ \neq 0 \Longrightarrow J \to J_c$ Canonical Coordinates

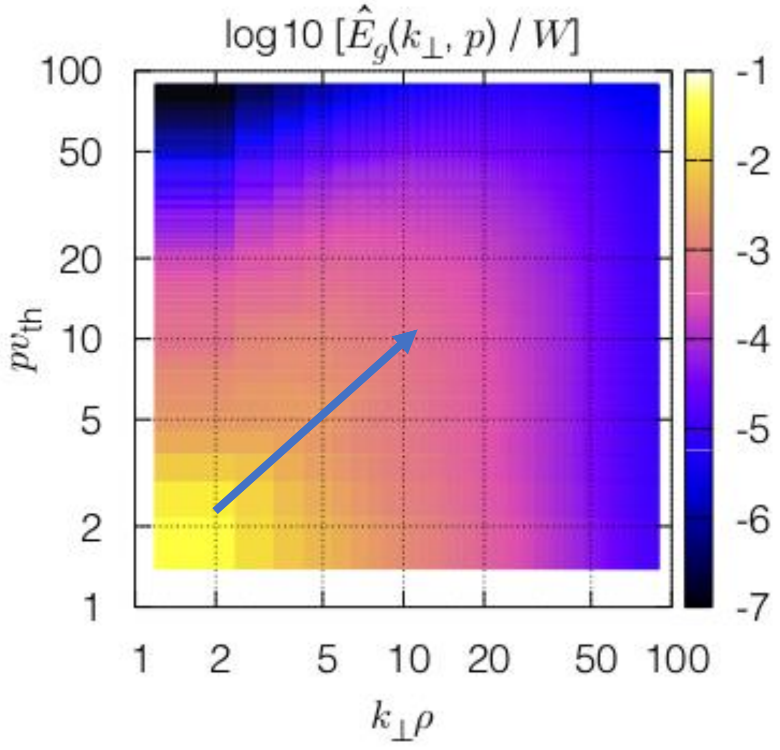
Sophus Lie: $detJ = 0 \Longrightarrow$ Canonical Coordinates plus Casimirs

$$J \to J_d = \begin{pmatrix} 0_N & I_N & 0 \\ -I_N & 0_N & 0 \\ 0 & 0 & 0_{M-2N} \end{pmatrix}.$$

Hamiltonian perspective of fluid and kinetic systems

Kinetic effects on fluid Casimirs (R. Numata)





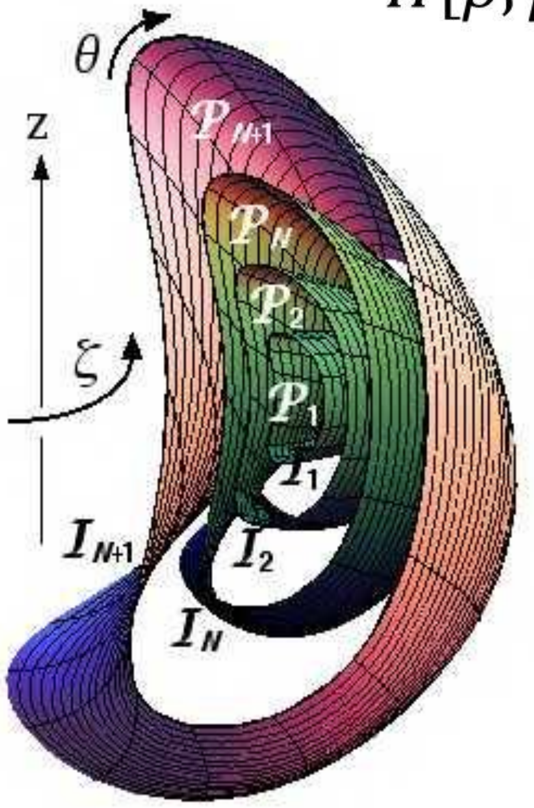
by T. Tatsuno

Modified Hamiltonian system (1)

Multi-region *relaxed* MHD (R.L. Dewar)

$$H[\rho, p, \mathbf{u}|\tau, \mu, \nu] = \int_{\Omega} \left(\rho \frac{u^2}{2} + \frac{p}{\gamma - 1} + \frac{B^2}{2\mu_0}\right) dV$$

$$-\tau S[\rho, p] - \mu K[\mathbf{A}] - \lambda K^{\mathrm{X}}[\mathbf{A}, \mathbf{u}]$$



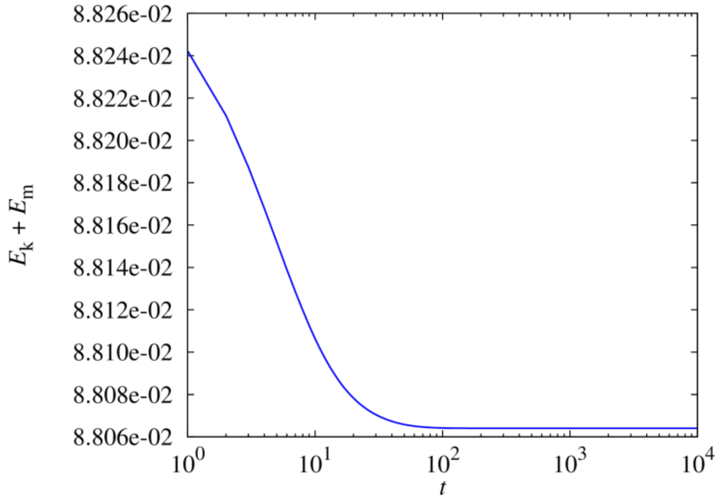
Replace by "global" constants (Casimirs) in each region

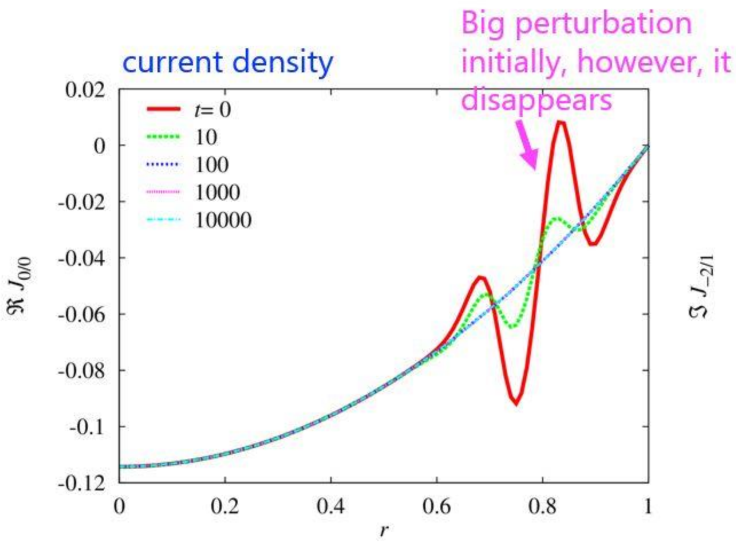
Modified Hamiltonian system (2)

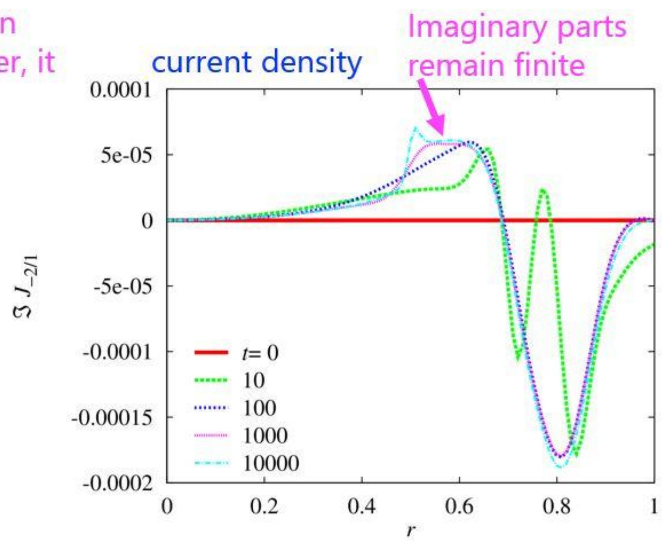
Modified Hamilton's equation for finding equilibrium solution (M. Furukawa)

$$\dot{u} = J\,\nabla H(u),\ J = \begin{pmatrix} 0 & I & 0 \\ -I & 0 & 0 \\ 0 & 0 & {\color{red}0} \end{pmatrix} \quad \Rightarrow \quad \dot{u} = J^2\,\nabla H(u),\ J^2 = \begin{pmatrix} -I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & {\color{red}0} \end{pmatrix}$$

energy relaxation with Casimir invariance → helical equilibrium







Theoretical approach to nonlinear phenomena/structures

Chaos

- “Matsuoka function” dictating HC-intersections of Henon map ← C. Matuoka
- Application of chaos for particle trap ← H. Saito

Nearly integrable dynamics

- Nonlinear Alfven wave and Recurrence ← R. Mukherjee (U30 winner)

Nonlinear processes in plasma

- Self-consistent nonlinear wave-particle resonances (oc trans.) ← Liu Chen
- Phase dynamics in 3-wave coupling ← C. Meng
- Deterministic nature of intermittency in geodesic acoustic mode ← Z. Liu
- Nonlinear saturation of SAW instability by kinetic effects ← T. Wang
- Carrier-envelope phase in nonlinear Compton scattering ← J.-X. Li

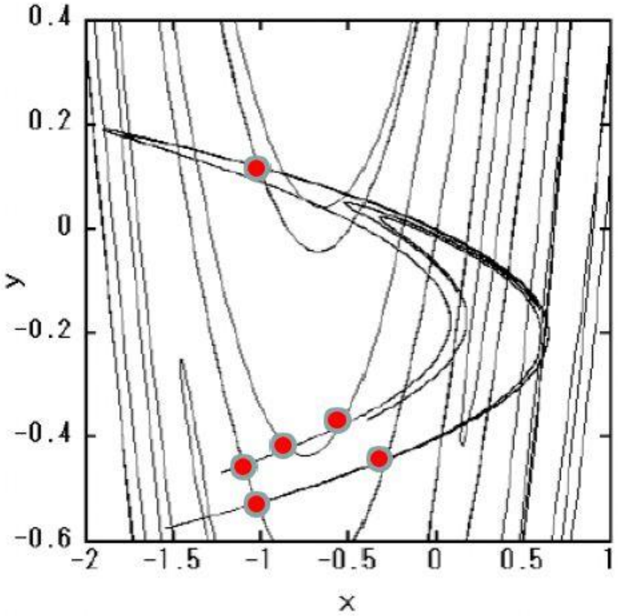
Theoretical approach to nonlinear phenomena/structures (1)

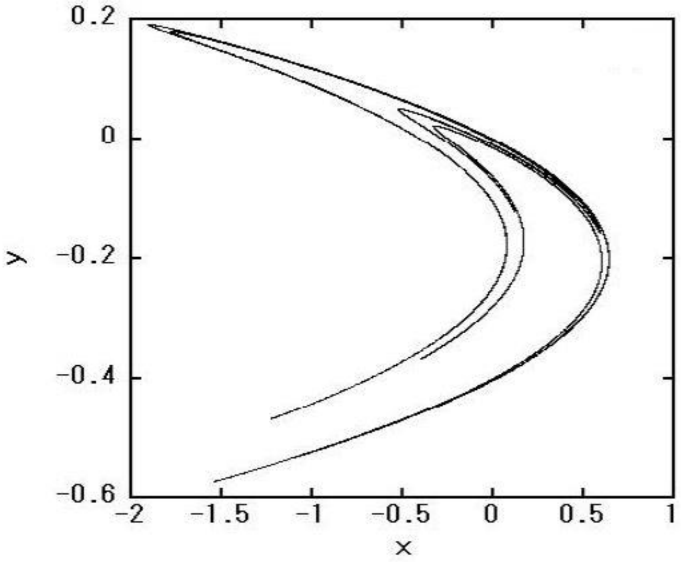
Analytilcal representation of chaotic attractors (C. Matsuoka)

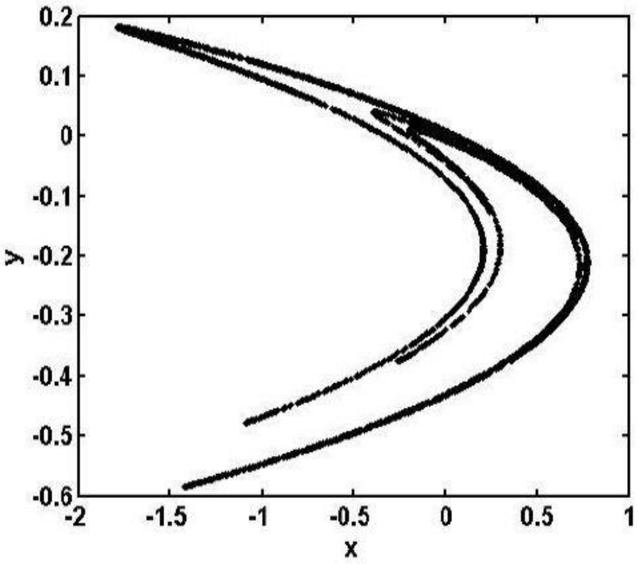
$$f : \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \mapsto \begin{pmatrix} x(t+1) \\ y(t+1) \end{pmatrix} = \begin{pmatrix} 1 + y(t) - ax(t)^2 \\ bx(t) \end{pmatrix}$$

$$x(t+1) - \lambda x(t) - bx(t-1) \;=\; -a\{x(t)\}^2$$

$$x(t) \;=\; \sum_{N=1}^{\infty} \frac{b_N (N-1)! e^{N\zeta_1 t}}{(t+n_0)^N} \left[1 + O(|t+n_0|^{-1})\right]$$

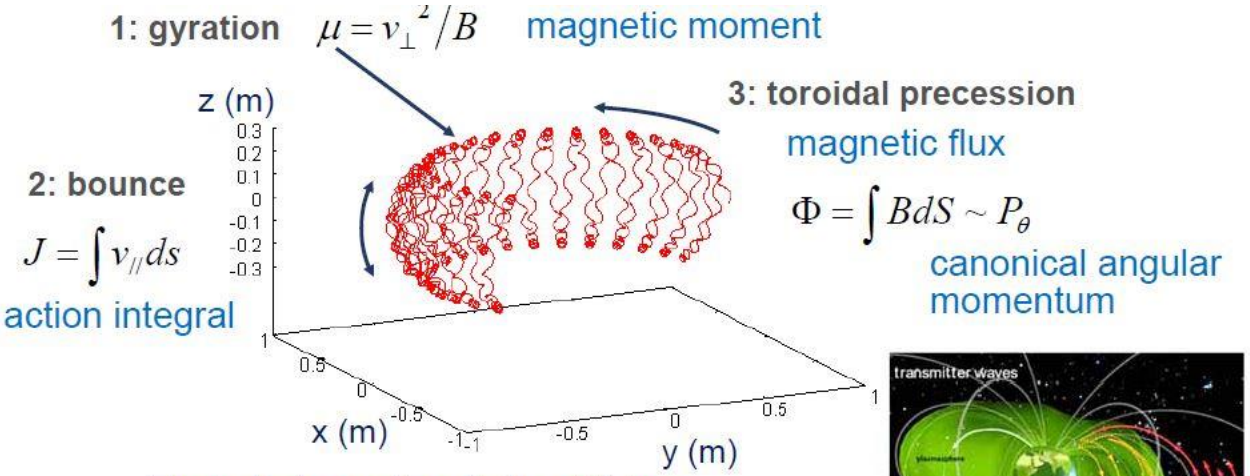




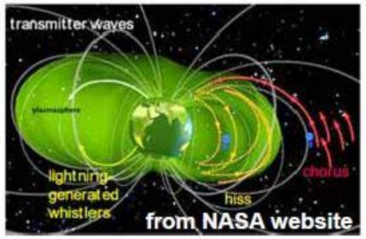


Theoretical approach to nonlinear phenomena/structures (2)

Chaotic motion of particles in magnetospheric system (H. Saitoh)

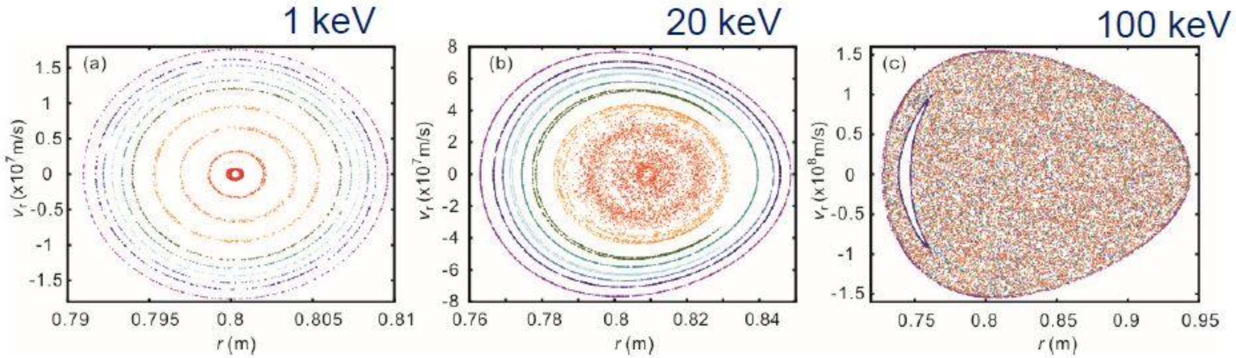


Typical charged particle orbit in a dipole magnetic field



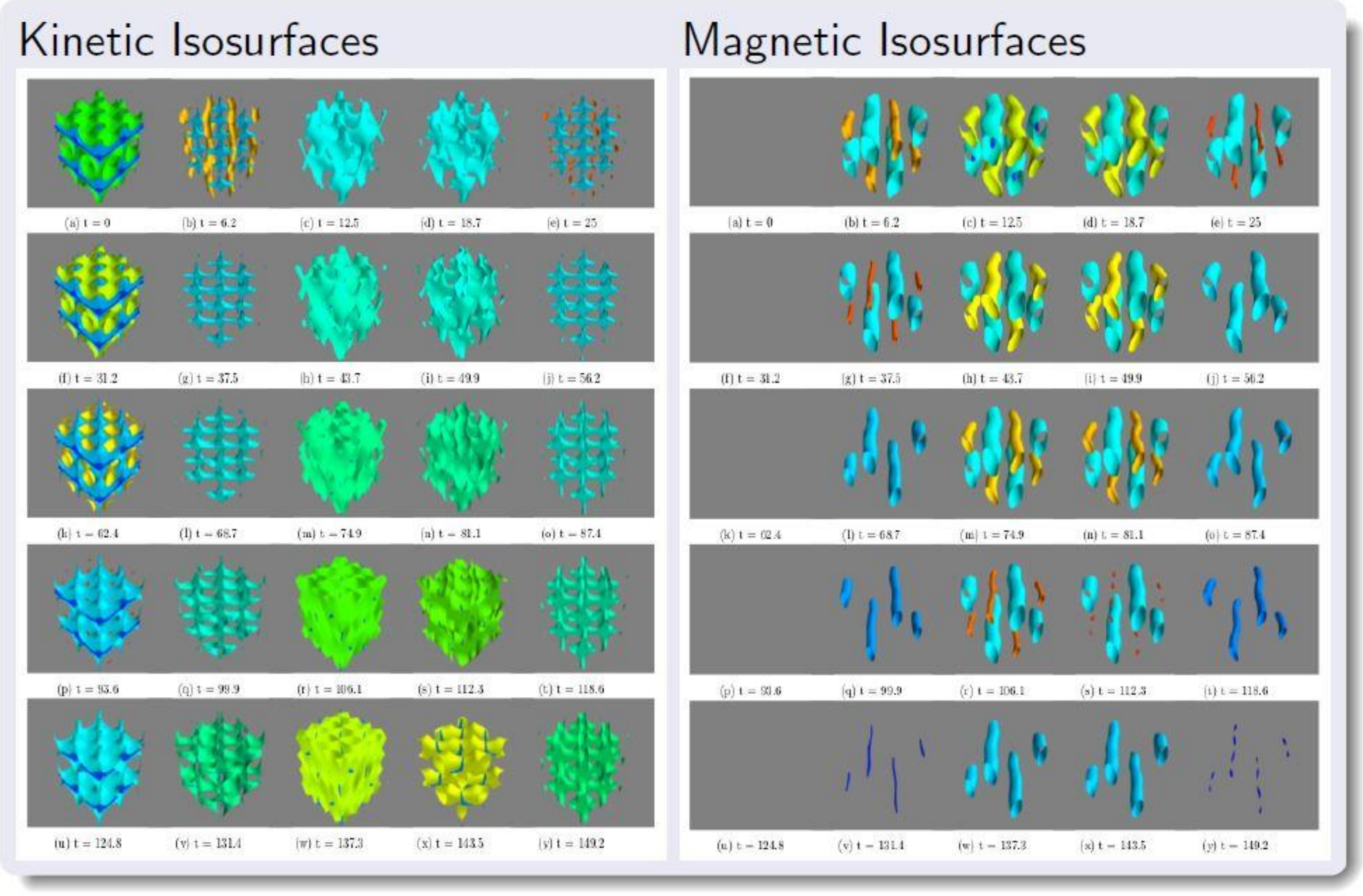
Scale separation is overcome for high-energy particles

Chaotic motion
→ effective injection



Theoretical approach to nonlinear phenomena/structures (3)

Recurrence in three dimensional magnetohydrodynamic plasma (R. Mukherjee)

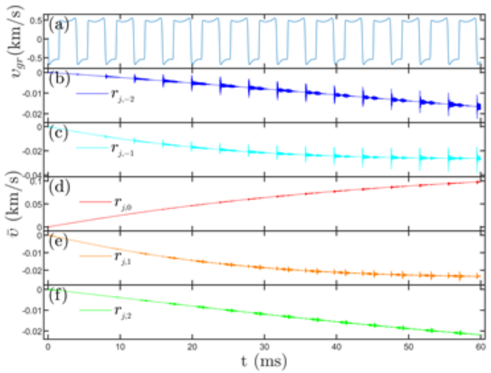


Nearly integrable structures by Energy-Casimir foliation

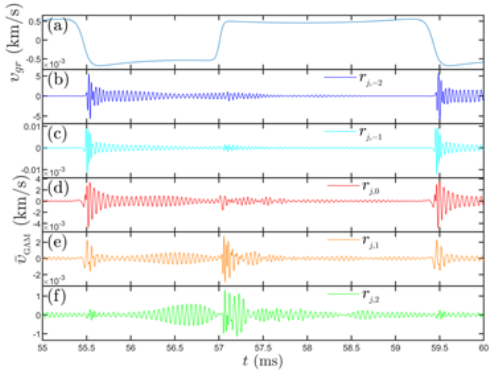
Theoretical approach to nonlinear phenomena/structures (4)

Deterministic nature of intermittency in geodesic acoustic mode (Z. Liu)

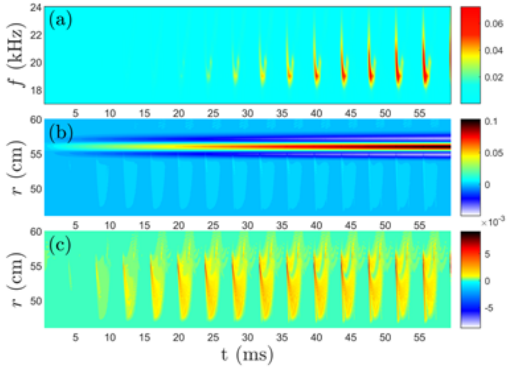
- Fig.1 Temporal evolution of radial group velocity and (b-f) zonal flow with intermittent excitation of GAM at five radial positions.



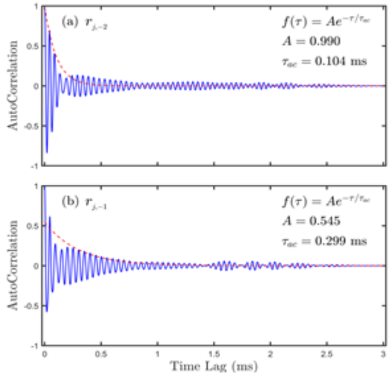
- Fig.2 The magnified graphs in one period (55-60ms) of Fig. 1 with low frequency portion filtered out.



- Fig.3 (a) Frequency-time spectrogram at $r_{j,0}$, (b) and (c) for spatiotemporal evolution for and GAM respectively



- Fig.4 Auto-correlation function for different radial position at (a) $r_{j,-1}$ and (b) $r_{j,-2}$



Challenge of scale hierarchy

Overcome multiscale difficulty by theoretical approach

- Small ε is good ! --- demonstrated by Ge-Fi with ***EB*** representation
  ← Liu Chen
- Geometrical reduction for kinetic model of "edge turbulence" ← N. Tronko
- Measuring multi-scale interactions ← S. Maeyama

Multi-scale

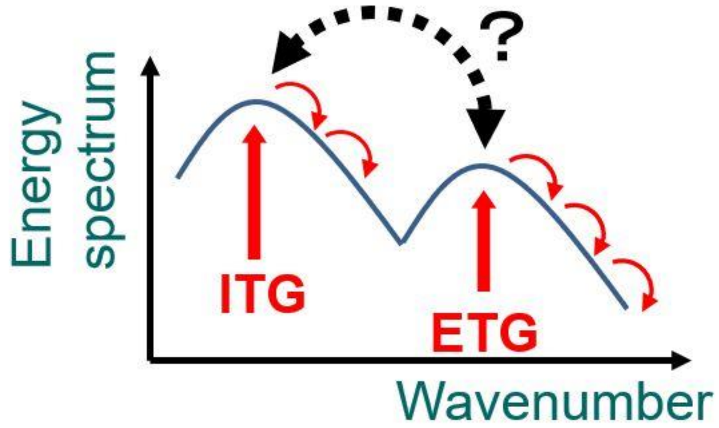
- High frequency mode generation by toroidal Alfven eigenmodes ← Z. Qiu

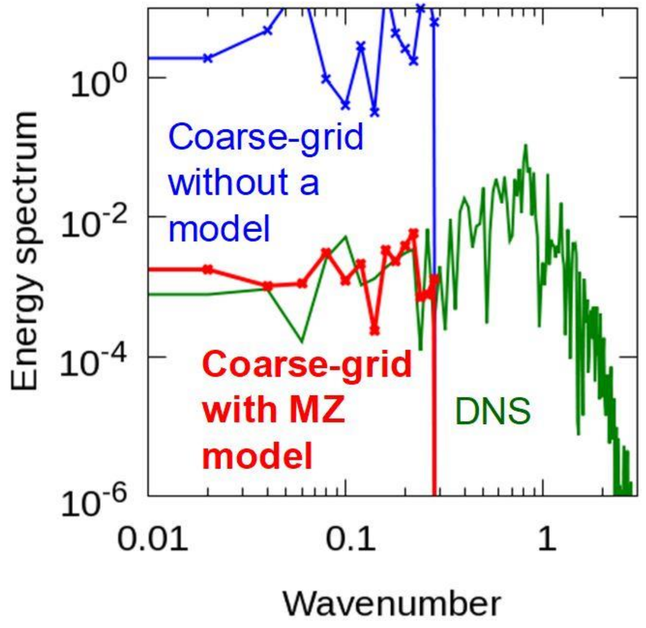
Renormalization

- N-particle → Vlasov by renormalization ← D.F. Escande

Challenge of scale hierarchy (1)

Interaction between different scales (S. Maeyama)





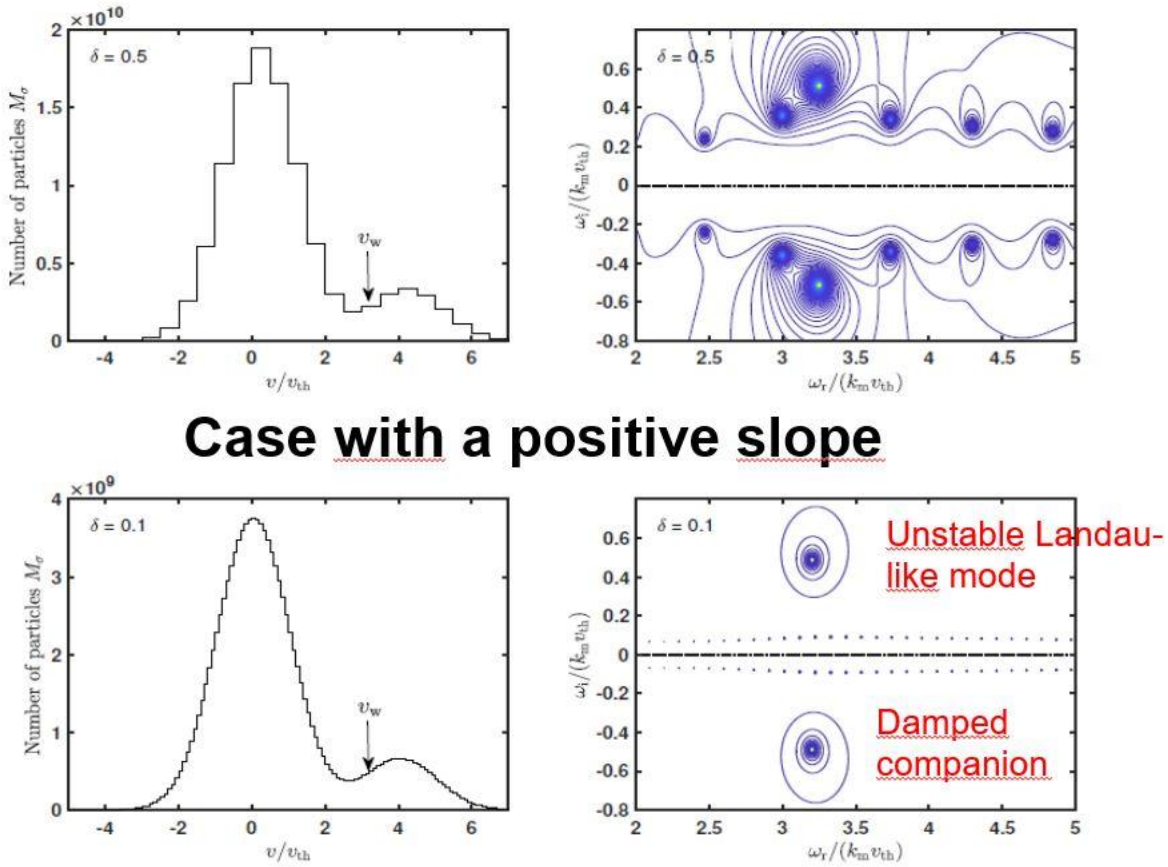
Generalized Langevin and Mori-Zwanzig projection

$$\hat{f}_k(t) = \Omega_k \hat{u}_k(t) - \int_0^t \Gamma(s) \cdot \boldsymbol{u}(t-s)ds + \hat{r}_k(t)$$

Small-scale effect | Correlated with large scale (~Coherent drag with memory) | Un-correlated with large scale (~Stochastic forcing)

Challenge of scale hierarchy (2)

From N-particle to Vlasov (D.F. Escande)



Renormalized particle
= Debye shielded particle

MHD equilibrium as a mathematical challenge

Existence of solutions

- Non-symmetric solutions ← N. Sato
- Construction by “annealing” ← M. Furukawa

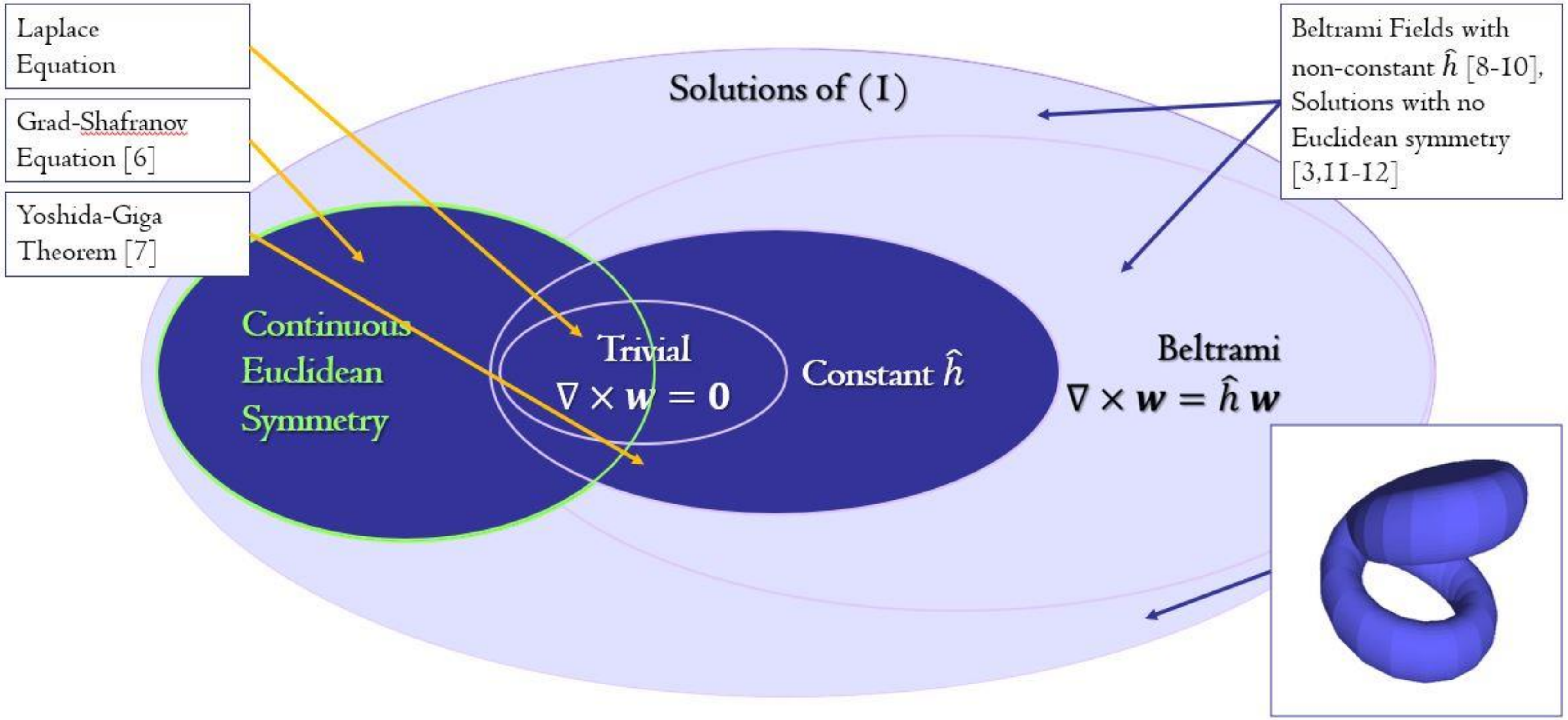
Solutions with singularities

- MRxMHD ← R.L. Dewar
- With flow ← Z. Qu
- Beltrami solver applying EM eigenfunctions ← D. Malhotra
- Current singularities ← Y. Zhou

MHD equilibrium as a mathematical challenge (1)

$$\boldsymbol{w} \times (\nabla \times \boldsymbol{w}) = \nabla \chi, \qquad \nabla \cdot \boldsymbol{w} = 0 \quad \text{in} \quad \Omega.$$

Although looks simple,
one of the most difficult PDE



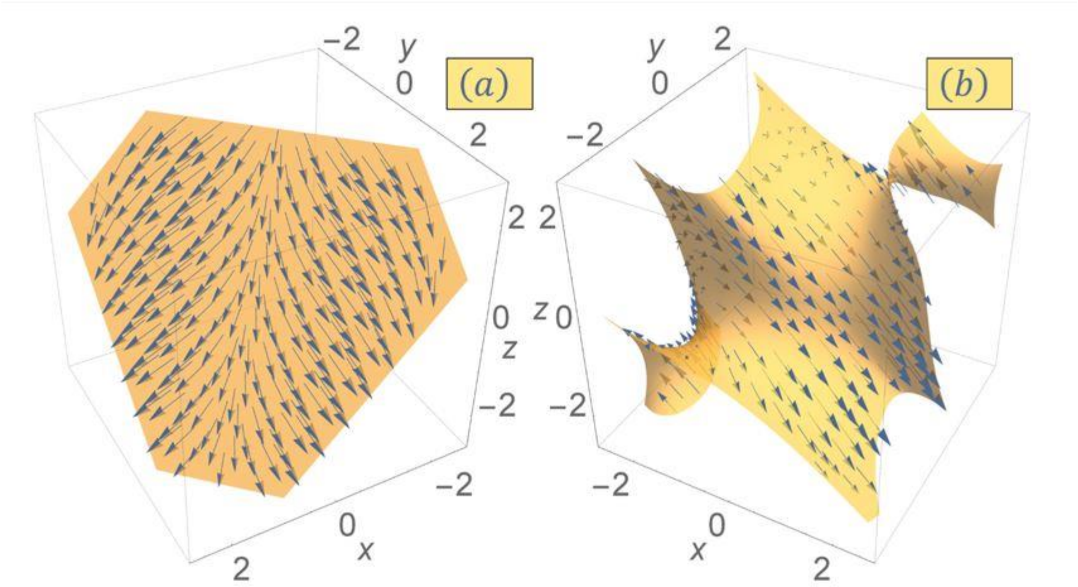
by N. Sato

MHD equilibrium as a mathematical challenge (2)

Construction of "non-symmetric" equilibria (N. Sato)

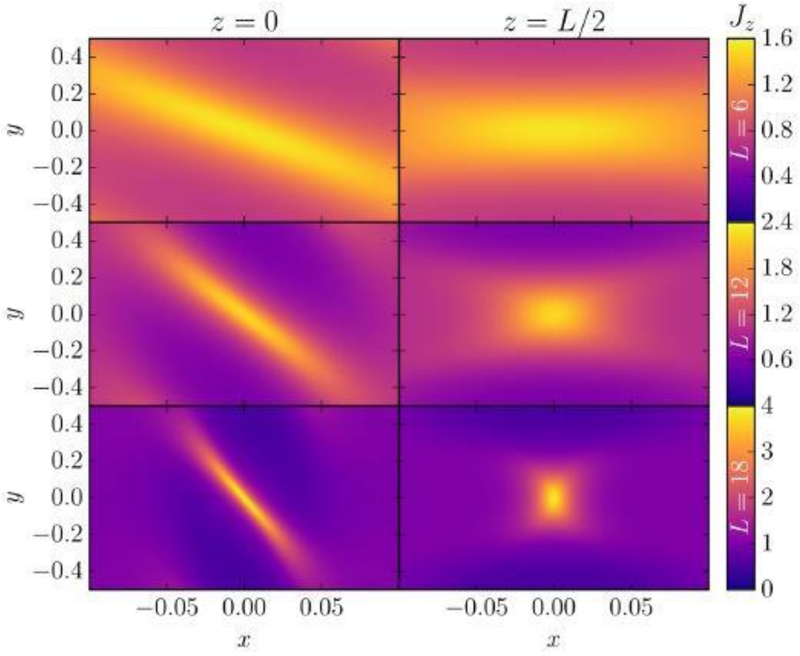
$$\boldsymbol{w} \times (\nabla \times \boldsymbol{w}) = \nabla \chi, \qquad \nabla \cdot \boldsymbol{w} = 0 \quad \text{in} \quad \Omega.$$

$$\boldsymbol{w} = \cos\theta\, \nabla\psi + \sin\theta\, \nabla\ell$$

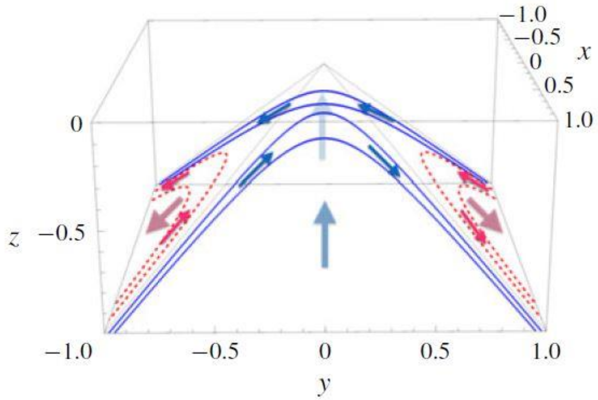


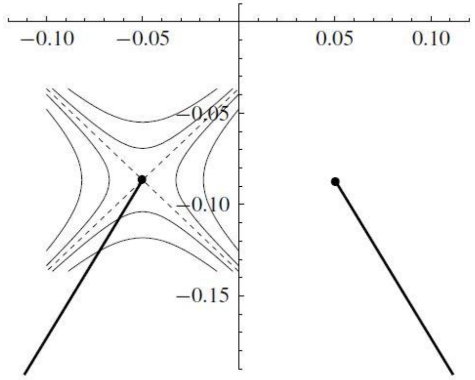
MHD equilibrium as a mathematical challenge (3)

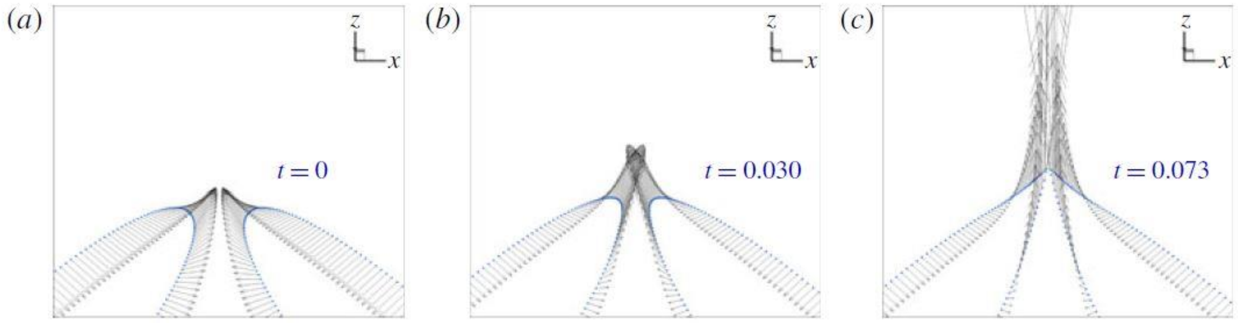
Current singularities generated in ideal MHD (Y. Zhou)



Vortex tube reconnection by Kimura:
finite time singularity?







Future

- Innovation in methodology for *representing* complexity.
    - topological / geometrical
    - data driven approach
    - clear description of scale hierarchy
- Rigorous proofs for conjectures
    - finite-time singularity
    - MHD equilibrium without symmetry
- Make good conjectures to attract young generation